Parameter Study of the 2D-Navier-Stokes-α Model

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In the late 1960's, Kraichnan, Leith and Batchelor (KLB)[1,4,6] proposed the existence of the two inertial regions in the energy spectrum of a two dimensional turbulent flow, namely the inverse energy cascade and the direct enstrophy cascade. Using dimensional analysis, KLB predicted that if energy is injected into a system at a rate ε at wavenumber k_f (see figure 1), then the energy spectrum in the energy cascade region (wavenumbers less than k_f) has the form $E(k) \propto \varepsilon^{2/3}k^{-5/3}$. Similarly, it can be shown using simple dimensional analysis that the energy spectrum in the enstrophy subrange (wavenumbers greater than k_f and less than the wavenumber k_d where viscous dissipation starts) has the form $E(k) \propto \eta^{2/3}k^{-3}$, where η is the forward enstrophy transfer rate to higher wave numbers.

The goal of this research study is to explore the practicality and the effectiveness of a particular model, the NS-α (pronounced NS-alpha) model, as a subgrid model to 2D-turbulence. To be more precise, we would like to compare the statistical behavior observed in the computations of NS-α model to the statistical behavior of the direct numerical simulation (DNS) of Navier-Stokes equations and see if we can recover KLB theory more clearly under a coarser grid.

To obtain our results we would generate the DNS of the Navier-Stokes equation at the largest resolution as computationally available. At the moment of this writing, even very powerful supercomputers have limited capabilities in resolving DNS due to wide range of scales of motion that must be computed. The simulation has to take into account the biggest scales of motion and at the same time be able to resolve the smallest scales of motion. In particular, we need to be able to store information at every grid point. This is in fact one of the main challenges of DNS. As an example, for 8192^2 simulation, using floating point calculations, storing the grid information requires

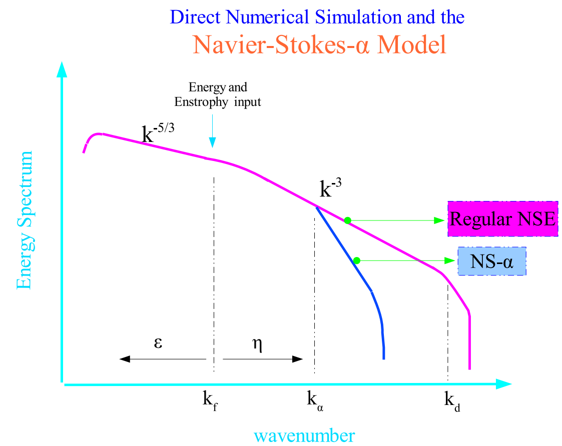

Figure 1: What we hope to see the behavior of the NS-α Model

the computer to have 1.5 GB of memory. This maybe the standard nowadays. However, for a 16384^2 simulation, we will need 4 GB of memory.

We normally need to atleast double the resolution in order to see an observable change in the slope of the energy spectrum in the enstrophy range. See for example figure 2. The higher the resolution, the closer we get to KLB theory. We need a high resolution in order to generate a longer enstrophy inertial range. The k^{-3} prediction (with some log correction see[5]) as the scaling power in the enstrophy subrange assumes that there is sufficient range of wavenumbers between the forcing wavenumber k_f and the wavenumber k_d where energy starts to be dissipated. Thus, if say we see a spectral slope of -3.2 in the enstrophy subrange in the 8192^2 simulation and we want to get even more closer to the k^{-3} as well as have a longer inertial range, then, we need to jump to atleast 16384^2 simulation. Another factor is the time required to resolve large simulations, for example we will need at least 16 cpus in order to get results at a reasonable time scale of days for a resolution of 4096^2. Clearly, DNS computation is not easily accesible. We hope the Navier-Stokes-alpha model will give us accurate statistics of the flow in the inertial range with less computational effort.

In this modeling scheme, we expect to retain the statistics of the larger scales of motion of lengthscale greater than α and at the same time prevent the creation of ever smaller scales of motion of lengthscale smaller than α. Figure 1 illustrates what we expect to see in the energy spectrum for the Navier-Stokes-alpha model. We expect that the statistics of the flow for scales of motion greater than alpha will not deviate from that of the statistics of the flow for Navier-Stokes equation. Then, for scales of motion smaller than alpha, i.e for wavenumbers greater than k_α, we expect the energy spectrum of the Navier-Stokes-alpha model to decrease faster than the Navier-Stokes equation. This, we hope, will give us clear statistics of the enstrophy range at a more resonable resolution since we don't have to resolve as much of the smaller scales.

Our preliminary results under a reasonable resolution of 4096^2 indicates a scaling of approximately $k^{-3.5}$ in the enstrophy subrange. We would like to compare what the NS-α does to the energy spectrum when the parameter α is adjusted so that it is in the enstrophy range with a reasonable distance from the wavenumber where the force is injected (k_f) and from the wavenumber k_d where the viscous term starts to dominate the nonlinear term.

Due to unavailability of computer resources we were able to produce only a partial result of the energy spectrum for the 4096^2 run. The spectrum given in figure 2 for 4096^2 is at a very early stage where the flow is not yet statistically steady. This is the steepest spectrum that we have observed so far. As the flow reaches the steady state, the spectrum can only get less steeper in the enstrophy subrange. As soon as the simulation reaches a steady state, we can stop the simulation and run the DNS of the the NS-alpha model with the same input parameters, with alpha nonzero. As an aside, the Navier-Stokes equations correspond to the NS-alpha model with alpha = 0.

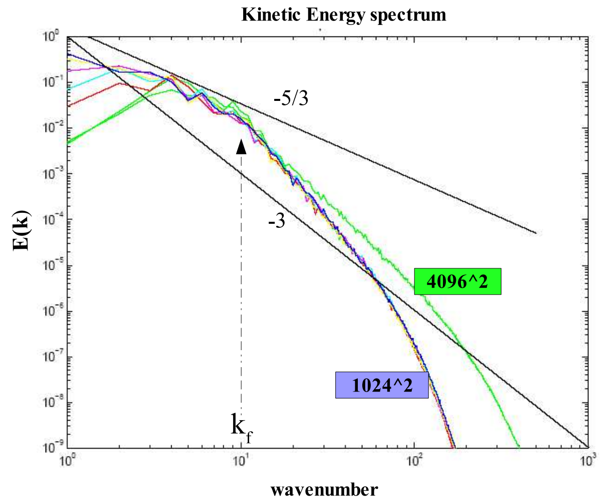

Figure 2: Log-log plot of the kinetic energy spectrum. A 1024^2 *run vs* 4096^2 *run (green spectrum). The forcing wavenumber is at* $k_f = 10$. *The two black lines are of slope -5/3 and -3. The* 1024^2 *resolution indicates approximately a scaling of* k^{-4} *and the* 4096^2 *resolution a scaling of* $k^{-3.5}$

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